Math 242 Midterm 1

Name: ______________________________

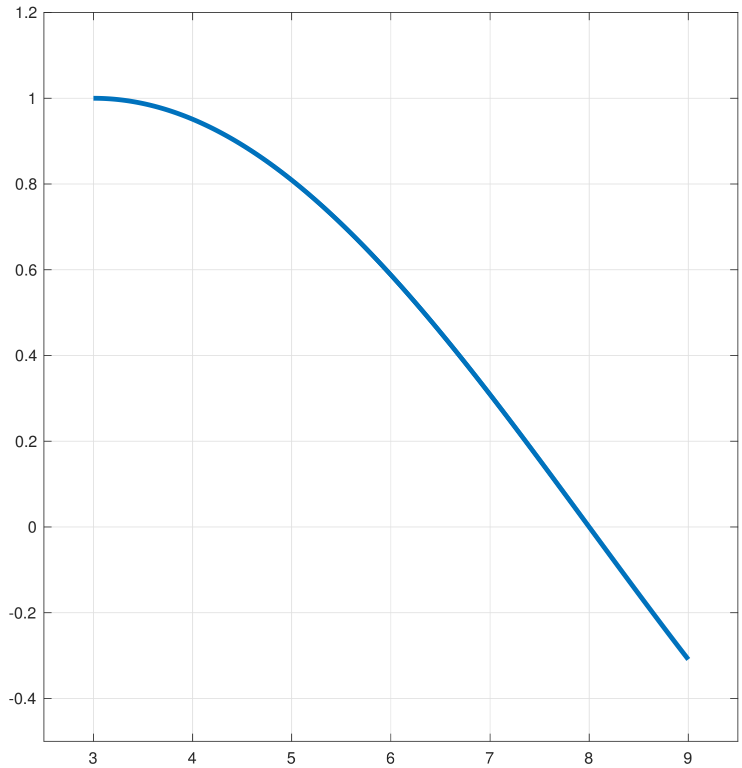
Please circle your section:

Recitation 1	Thurs 12-12:50	TA - Dan Flores
Recitation 2	Thurs 1:30-2:20	TA - Dan Flores
Recitation 3	Tues 9-9:50	TA - Vince Chung
Recitation 4	Tues 12-12:50	TA - Vince Chung
Recitation 5	Wed 9:30-10:20	TA - Lance Ferrer
Recitation 6	Wed 12:30-1:20	TA - Lance Ferrer
Recitation 7	Fri 10:30-11:20	TA - Ikenna Nometa
Recitation 8	Fri 12:30-1:20	TA - Ikenna Nometa
Recitation 9	Fri 9:30-10:20	TA - Dan Flores

Question	Points	Score
1	12	
2	8	
3	14	
4	10	
5	16	
6	40	
Total:	100	

- You may not use notes or calculators on the test.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it).
- The last page is a formula sheet. You are welcome to remove this from the exam.
- Good luck!

1. Given below is the graph of a one-to-one function f whose domain is the interval $[3, 9]$.



(a) (4 points) Determine $f^{-1}(0.8)$. [Approximately, if need be.]

(b) (4 points) Which of the following values is closest to $\frac{df^{-1}}{dx}(0.6)$? (Be aware of the scaling of the axes!)

1. $\frac{df^{-1}}{dx}(0.6) = -5$
2. $\frac{df^{-1}}{dx}(0.6) = -1/5$
3. $\frac{df^{-1}}{dx}(0.6) = 0$
4. $\frac{df^{-1}}{dx}(0.6) = 1/5$
5. $\frac{df^{-1}}{dx}(0.6) = 5$

(c) (4 points) Which of the following intervals most closely resembles the domain of f^{-1}?

1. $[-0.3, 1]$
2. $[1, 1.3]$
3. $[3, 9]$
4. $(-\infty, \infty)$

2. Find the exact value:

(a) (4 points) $\log_4 20 - \log_4 5$

(b) (4 points) $\cot\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

3. Differentiate with respect to x. You do not have to simplify your answers.

(a) (7 points) $y = \sin\left(e^{-2x}\right)$

(b) (7 points) $y = (\ln(x))^{\ln(x)}$

4. (10 points) The population of an idealized colony of bacteria grows exponentially, so that the population doubles every half-hour. The experiment begins at 6:00pm. If at 6:10pm the population is measured at 20 bacteria, how many will there be at 8:00pm?

5. Find the following limits. Remember to use proper notation, and to indicate if you are using L'Hospital's Rule.

(a) (8 points) $\lim_{x \to -\infty} \arctan\left(\frac{1+x}{3-x}\right)$

(b) (8 points) $\lim_{x \to 0} \frac{\cos(5x) - 1}{x \sin(2x)}$

6. Evaluate the following integrals

   (a) (10 points) $\int xe^{3x}\mathrm{d}x$

(b) (10 points) $\displaystyle\int \frac{\mathrm{d}x}{x^2\sqrt{x^2+1}}$

(c) (10 points) $\int \cos^2(4x)\mathrm{d}x$

(d) (10 points) $\displaystyle\int_2^3 \frac{\left(\ln(x^2)\right)^2}{x}\,\mathrm{d}x$

Formula sheet

- Derivatives of inverse trigonometric functions.

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{x\sqrt{x^2-1}}$$

- Trigonometric identities.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$

$$\sin x \cos x = \frac{1}{2}\sin(2x)$$

$$\sin x \sin y = \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y)$$

$$\cos x \cos y = \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y)$$

$$\sin x \cos y = \frac{1}{2}\sin(x-y) + \frac{1}{2}\sin(x+y)$$

- Integrals of trigonometric functions.

$$\int \tan x \; dx = \ln|\sec x| + C$$

$$\int \cot x \; dx = \ln|\sin x| + C$$

$$\int \sec x \; dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \; dx = -\ln|\csc x + \cot x| + C$$